① $\frac{dy}{dx} + y\log y = xye^{x}$

Soln. $\frac{dy}{dx} + y\log y = xye^{x}$

Dividing throughout by xy, we have

$\frac{1}{y}\frac{dy}{dx} + \frac{\log y}{x} = e^{x}$ ∴ Put $\log y = v$

$\therefore \frac{dv}{dx} + \frac{v}{x} = e^{x}$ — ① $\Rightarrow \frac{dy}{y} = dv$

It is linear Diff. eqn. Here, $P = \frac{1}{x}$

$\Rightarrow \int P dx$

$= \int \frac{1}{x} dx = \log x$

$\therefore$ I.F. $= e^{\log x} = x$

Multiplying ① by I.F., we have

$$x\frac{dv}{dx}+v=xe^{x}.$$

on integrating both sides, we have

$$xv=\int xe^{x}dx+c$$

$$=xe^{x}-e^{x}+c$$

$$\Rightarrow x\log y=e^{x}(x-1)+c$$

required Solution

(2) $(x^3y^3+x^2y^2+xy+1)y\,dx+(x^3y^3-x^2y^2-xy+1)x\,dy=0$

Soln- We have

$$(x^3y^3+x^2y^2+xy+1)y\,dx+(x^3y^3-x^2y^2-xy+1)x\,dy=0$$

$$\text{or}\ (x^2y^2+1)(xy+1)\cdot y\,dx+(x^2y^2-1)(xy-1)x\,dy=0 \quad \text{(1)}$$

It is of the form.

$$f_1(x\cdot y)y\,dx+[f_2(x\cdot y)]x\,dy=0.$$

Now, $Mx-Ny$

$$=(x^3y^3+x^2y^2+xy+1)xy-(x^3y^3-x^2y^2-xy+1)xy$$

$$=2x^2y^2(xy+1)\neq 0.$$

$$\therefore \text{I.F.}=\frac{1}{2x^2y^2(xy+1)}$$

Multiplying (1) by I.F., we have

$$\frac{1}{2}\frac{(x^2y^2+1)}{x^2y^2}y\,dx+\frac{(xy-1)^2}{2x^2y^2}x\,dy=0$$

$$\text{or}\ \frac{1}{2}\frac{x^2y^2+1}{x^2y^2}(y\,dx+x\,dy)-\frac{2xy}{2x^2y^2}x\,dy=0$$

$$\text{or}\ \frac{1}{2}\left\{1+\frac{1}{(xy)^2}\right\}d(xy)-\frac{dy}{y}=0$$

On integrating, we have

$$\frac{1}{2}\left(xy - \frac{1}{xy}\right) - \log y = \log c$$

where c is the integration Constt.

(3) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\sin x$

Soln. Let $y = e^{mx}$ be the solution of the above eqn.

Hence, Auxiliary eqn. is given by

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1$

$m = 1$

$\therefore m = 1$ is repatated twice.

$\therefore C.F. = (A_1 e^x + A_2 x e^x)$

$= (A_1 + A_2 x) e^x$

Now, $P.I = \frac{1}{(D-1)^2} x \sin x$

$= R.P. \frac{1}{(D-1)^2} x e^{ix}$ $= \frac{1}{D^2 - 2D + 1} x\sin x$

$\cancel{R.P. e^{ix} \frac{1}{(D-i-1)^2} x}$ $= \frac{1}{(D+1)^2 - 2(D+1) + 1} x$

$\cancel{R.P. e^{ix} \frac{1}{D^2 + i^2 + 1 - 2Di - 2D -}}$

$= \frac{1}{D^2}(x\sin x) = \frac{1}{D}\int x\sin x\,dx$

$= \frac{1}{D}\left[-x\cos x - \int -\cos x\,dx\right]$

$= \frac{1}{D}\left[-x\cos x + \sin x\right]$

$= \left[-x\sin x - \int -\sin x\,dx - \cos x\right]$

$= [-x\sin x - \cos x - \cos x]$

$= [-x\sin x - 2\cos x]$

$= -(x\sin x + 2\cos x)$

$\therefore$ Complete Solution, $y = C.F. + P.I.$

$y = (A_1 + A_2)e^{x} - (x\sin x + 2\cos x)$